

RELAXATION METHOD APPLIED TO NETWORK PROBLEM INVOLVED IN ELECTRIC RAILWAY SYSTEM

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ABSTRACT. In this paper it has been shown how the network problem involving electric railway systems, can be solved using relaxation technique. The present method utilises the set of linear simultaneous equations which are obtained with the help of Kirchhoff's laws of electrical network, and shows how to solve them. The relaxational solution as obtained is seen to be quite useful because it gives the values of the unknown voltages at all the nodal points simultaneously.

INTRODUCTION

In the electric railway systems (Starr, 1946), a definite electrical problem is difficult to be formulated due to the fluctuating loads. But assuming the loads simulating the typical operating conditions the problem can be solved by different methods, which entail much more labour with the increase of nodal points in the corresponding network. But the method discussed in this paper shows its advantage in the sense that the increase in the number of nodal points does not generally bring about more complication in solving the problem.

In this network system as shown in Fig. 1 the supply voltages, resistances of the trolley, feeders and rails, and ampere loads at the designated locations are known. The equivalent circuit diagram can be drawn as in Fig. 2. Considering the nodal points of the Fig. 2, a set of linear simultaneous equations can be obtained at each of them applying Kirchhoff's laws of networks and these equations are then solved by relaxation method.

THE METHOD

In this method the following linear simultaneous equations are obtained if the required nodal points of the equivalent circuit diagram are considered.

Hence :

$$\begin{aligned}
 \text{At } A, & I_A - (V_A - V_B)g_{AB} - (V_A - V_D)g_{AD} - (V_A - V_C)g_{AC} = 0 \\
 C, & (V_A - V_C)g_{AC} - (V_C - V_B)g_{CB} - (V_C - V_D)g_{CD} = 0 \\
 B, & (V_A - V_B)g_{AB} + (V_C - V_B)g_{CB} - I_{BB'} = 0 \\
 D, & (V_A - V_D)g_{AD} + (V_C - V_D)g_{CD} - I_{DD'} = 0 \\
 B', & I_{BB'} + (V_C' - V_{B'})g_{C'B'} - (V_{B'} - V_{A'})g_{B'A'} = 0 \\
 D', & I_{DD'} - (V_{D'} - V_{C'})g_{D'C'} - (V_{D'} - V_{A'})g_{D'A'} = 0
 \end{aligned} \tag{1}$$

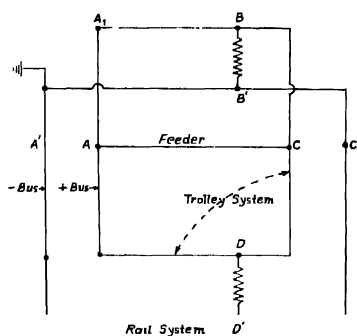


Fig. 1. Diagram for network in Electric Railway System

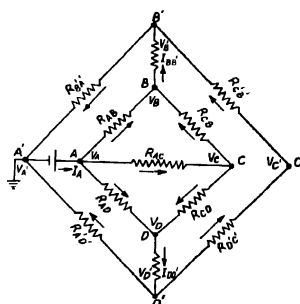


Fig. 2. Equivalent diagram for Network in Electric Railway System.

where

I_A is the current flowing towards A ,

$I_{BB'}$ through the branch BB' ,

$I_{DD'}$ DD' ,

V_A potential at the nodal point A ,

V_B B ,

V_C C ,

V_D D ,

$V_{A'}$ $A' = \text{zero (earth point)}$

$V_{B'}$ B'

$V_{C'}$ C' ,

$V_{D'}$ D' ,

$g_{AB} = 1/R_{AB}$, where R_{AB} is the resistance of the branch AB ,

$g_{AC} = 1/R_{AC}$, R_{AC} AC ,

$g_{AD} = 1/R_{AD}$, R_{AD} AD ,

$g_{CB} = 1/R_{CB}$, R_{CB} CB ,

$g_{CD} = 1/R_{CD}$, R_{CD} CD ,

$g_{B'C'} = 1/R_{B'C'}$... $R_{B'C'}$ $B'C'$,

$g_{B'A'} = 1/R_{B'A'}$... $R_{B'A'}$ $B'A'$,

$g_{D'C'} = 1/R_{D'C'}$... $R_{D'C'}$ $D'C'$,

The above set of equations after necessary simplification and rearrangement can be written as shown below :

$$\begin{aligned}
 I_A - V_A(g_{AB} + g_{AD} + g_{AC}) + V_B g_{AB} + V_C g_{AC} + V_D g_{AD} &= 0 = F_1 \\
 V_A g_{AC} + V_B g_{CB} - V_C(g_{AC} + g_{CB} + g_{CD}) + V_D g_{CD} &= 0 = F_2 \\
 V_A g_{AD} - V_B(g_{AB} + g_{CB}) + V_C g_{CB} - I_{BB'} &= 0 = F_3 \\
 V_A g_{AD} - V_C g_{CD} - V_D(g_{AD} + g_{CD}) - I_{DD'} &= 0 = F_4 \\
 I_{BB'} - V_{B'}(g_{C'B'} + g_{B'A'}) + V_{C'} g_{C'B'} &= 0 = F_5 \\
 I_{DD'} - V_{D'}(g_{D'C'} + g_{D'A'}) + V_{C'} g_{D'C'} &= 0 = F_6
 \end{aligned} \tag{2}$$

where F_1, F_2, F_3, F_4, F_5 and F_6 are the residuals. The values of the unknowns shown in the relation (2) can be easily found out when the residuals are liquidated by relaxation method. To liquidate them the basic unit, block and group operations are carried out (Allen, 1954). In basic unit operation (Table I) the changes in the values of the residuals due to unit positive increment of the unknowns are found out. With the help of basic unit operations suitable block and group operations can be performed in which equal simultaneous, and unequal simultaneous increments are given respectively to more than one unknown to bring about the changes in some required residuals without affecting the rest. In the relaxation table (Table II) the use of basic, block and group operations are shown in the liquidation steps, number 2 and 6; 1 and 5, and 3, 4 and 7 respectively. The liquidation is nearly complete in those seven steps yielding the values of the unknowns. The following illustration will clearly show the merit and utility of the method.

ILLUSTRATION

This illustrating example described hereafter is solved by Dawes (1952) by conventional method.

In Fig. 1 there is shown a simple railway system with a ring connected trolley and a single feeder connected to the busbars at A and to the trolley system at C . The station busbars at AA' are maintained at 600 volts, busbar A being positive and A' being negative and grounded. The resistance of the busbar is negligible. The resistance of the overhead trolley is as follows : A to $B = 0.30$ ohm, B to $C = 0.20$ ohm, C to $D = 0.20$ ohm, D to $A = 0.28$ ohm. A feeder connected from A to C and its resistance is 0.25 ohm. The resistance of the rail and the ground return is as follows : A' to $B' = 0.40$ ohm, B' to $C' = 0.25$ ohm, C' to $D' = 0.25$ ohm, D' to $A' = 0.36$ ohm. A trolley car at BB' takes 70 amps and a car at DD' takes 80 amps. It was desired to determine (a) Current in each section of trolley and in feeder, (b) Voltages at each car and at feeding point CC' .

Considering the equivalent circuit diagram shown in Fig. 2 of the railway

TABLE I
Basic Unit Operation Table

δV_B	δV_C	δV_D	$\delta V_{B'}$	$\delta V_{C'}$	$\delta V_{D'}$	δF_1	δF_2	δF_3	δF_4	δF_5	δF_6
1	—	—	—	—	—	3.333	5.0	-8.333	0	0	0
—	1	—	—	—	—	4.600	-14.0	5.600	3.0	0	0
—	—	1	—	—	—	3.571	5.0	0	-3.571	0	0
—	—	—	1	—	—	0	0	0	0	-6.3	0
—	—	—	—	1	—	0	0	0	0	4.0	0
—	—	—	—	—	1	0	0	0	0	0	-6.778

 TABLE II
Relaxation Table

Liquidation steps	δV_B	δV_C	δV_D	$\delta V_{B'}$	$\delta V_{C'}$	$\delta V_{D'}$	F_1	F_2	F_3	F_4	F_5	F_6
	$V_B = V_C$	$V_C = V_D$	$V_D = V_{B'}$	$V_{B'} = V_{C'}$	$V_{C'} = V_{D'}$	$V_{D'} = 0$	-6302.4	2400	1030	262.6	70	80
(1)	356	2436	586	2436	586	2436	0	15.0256	-23.0400	-30.8750	70	80
(2)	—	—	—	—	—	—	15.2916	0	-4.2979	-11.2320	70	80
(3)	-0.5158	—	—	—	—	—	15.8444	0	0	-15.6448	70	80
(4)	-0.6859	-1.1433	-2.5133	—	—	—	0	0	0	6.10609	70	80
(5)	—	—	—	-28.0	28.0	28.0	0	0	0	6.10609	0	2.216
(6)	—	—	—	—	—	0.3269	0	0	0	6.10609	0	0
(7)	—	0.1	0.1	0.3	0.15	0.2731	0.7571	-0.9	0.3	-0.16011	-0.15	-0.05
Values of the unknown	585.042	589.121	584.344	28.3	28.45	28.60	0.7571	-0.9	0.3	-0.16011	-0.15	-0.05
												Residuals unliquidated

 TABLE III
Comparison of values

Methods	I_{AB}	I_{AD}	$I_{B'A'}$	$I_{D'A'}$	I_{AC}	I_{BC}	I_{DC}	$I_{D'C'}$	$I_{B'B'}$	$I_{D'D'}$	V_C	$V_{C'}$
	amps	amps	amps	amps	amps	amps	amps	amps	amps	amps	volts	volts
Relaxation method	49.86	55.91	70.75	79.44	43.48	20.44	23.93	0.60	0.60	536.74	557.74	589.13
Conventional method	30.00	56.20	70.60	79.40	43.80	20.00	23.80	0.60	0.60	536.80	557.60	589.10

or nodal points the network becomes complicated, with some practice this method needs practically no extra labour in solving the problem. In the relaxation table (Table II) the residuals are not liquidated completely and consequently their values are reduced to narrowest possible limits so that the desired limit of accuracy of the values can be achieved.

A C K N O W L E D G M E N T

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R E F E R E N C E S

- Allon, D. N. deG., 1951, *Relaxation Methods*, Chapter 1 and 2, (McGraw-Hill Book Co., Inc., New York).
- Duwez, G. L., 1952, *A Course in Electrical Engineering*, Vol. 1, (McGraw-Hill Book Co., Inc., New York), 76.
- Starr, A. T., 1946, *Generation, Transmission and Utilisation of Electrical Power*, (Sir Isaac Pitman & Sons Ltd., London) 344